Incremental Inductive Verification

slides adapted from the ones by
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Inductive Verification

In order to prove that $P(x)$ is $\mathcal{M}$-invariant, one possibility is to check if P is inductive. With 2 SAT-solver calls, we check the validity of:

$$\text{(initiation)} \quad I \Rightarrow P$$
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Q: Why?

Remember the discussion about k-induction we did on the 5^{th} lecture?

The set of states in P could be must larger than the ones in I and contain states unreachable from I via T.

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- **Incremental proof**: look for a sequence of lemmata $\phi_1, \phi_2, \ldots, \phi_k = P$ such that ϕ_i is inductive relative to $\phi_1 \wedge \cdots \wedge \phi_{i-1}$, for all $1 < i \leq k$, *i.e.*,
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  It follows that $P \wedge \bigwedge_{i=1}^{k-1} \phi_i$ is an inductive strengthening.

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Note that:

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Note that:

- both methods do *not* compute a formula R for the exact set of reachable states in $\mathcal{M}$;
- rather, they find a formula $F \wedge P$ that represents a larger set of states *all satisfying* $F \wedge P$:
  - $\Rightarrow$ this F is a much smaller formula than R.

Monolithic Approach - Naïve algorithm

Naïve algorithm for finding an inductive strengthening:

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At the end, the inductive strengthening (if any) will be:

$$P \wedge \bigwedge_{err \in CTI} \neg err$$

Incremental Proof - Example

Consider the program $\mathcal{M}_1$:

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x,y := 1,1
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Q: why?

We are not saying anything about x.

Incremental Proof - Example

We establish the first inductive incremental lemma $\phi_1 \coloneqq x \geq 0$:

- $x = 1 \wedge y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow x' \geq 0$

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We have found the inductive strengthening $\phi_1 \land \phi_2$, by means of an incremental proof.

Limitation of incremental proofs

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Monolithic approach = worst case of incremental proofs.

FSIS - Algorithm

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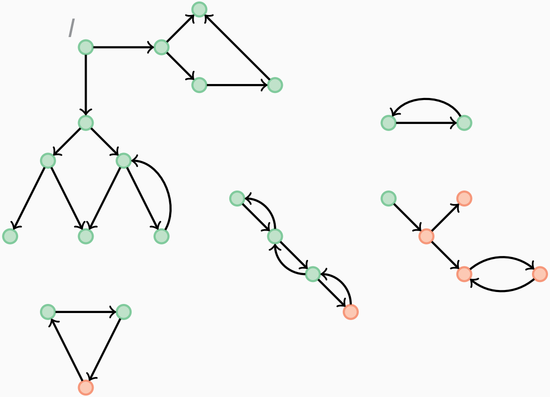
- FSIS: Finite-State Inductive Strengthening. It follows the incremental methodology [BM07].
- “this algorithm is a result of asking the question: if the incremental method is often better for humans, might it be better for algorithms as well?" [Bra12];
- the core of the algorithm is the generalization of an error state.

FSIS - Example



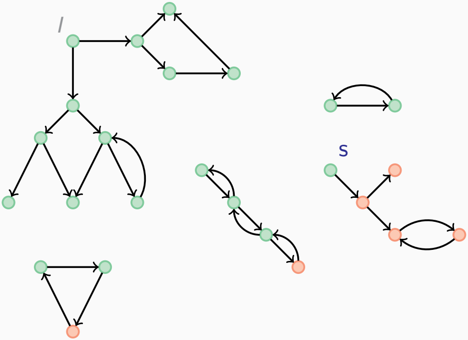
FSIS - Example - 1^{st} iteration

Check if P is inductive (relative to nobody). Check the validity of:

✓ $I \Rightarrow P$

✗ $P \wedge T \Rightarrow P'$

State s is a CTI.



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  - $\phi_1 \subseteq \neg s$; *(it excludes s)*
  - ϕ_1 is inductive (relative to its own); *(it includes at least all the reachable states)*
  - ϕ_1 is minimal. *(it excludes the maximal number of non-reachable states)*

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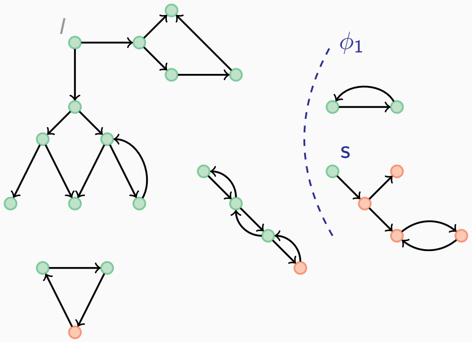
- if ϕ_1 does exist, it becomes the first incremental lemma.

FSIS - Example - 1^{st} iteration

ϕ_1 can be thought as a "boolean" cutting plane.



Which states are excluded by ϕ_1? (i) those who can reach s
(ii) states “similar" to s (they share with s the dropped literals).

FSIS - Example - 2^{nd} iteration

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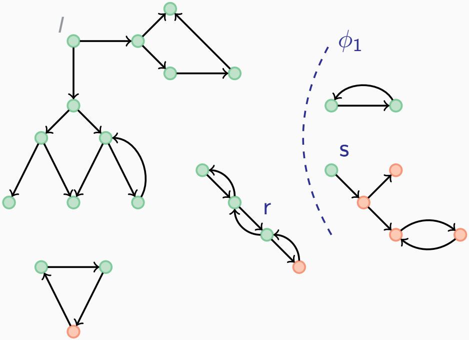

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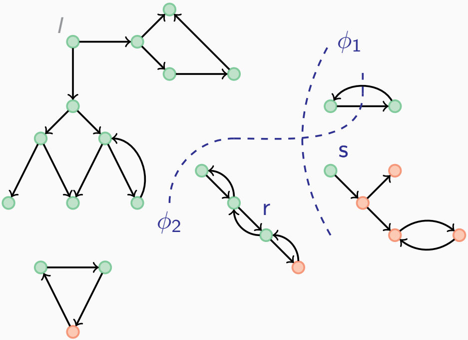

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Why "inductive relative to"?

- it would have been correct to generate ϕ_2 inductive (relative to its own), but it's more than what we need;
  - at the end we will consider the AND of all the lemmata;

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Why "inductive relative to"?

- it would have been correct to generate ϕ_2 inductive (relative to its own), but it's more than what we need;
    - at the end we will consider the AND of all the lemmata;
- in general, it is faster to generate "inductive relative to" clauses.
    - intuitively, we are considering many fewer states of the system.

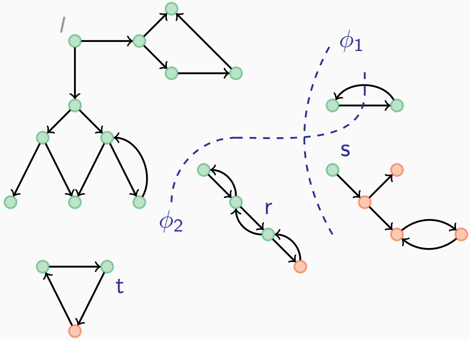
FSIS - Example - 3^{rd} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge P \wedge T \not\Rightarrow P'$

State t is a CTI.



FSIS - Example - 3^{rd} iteration

- generalization of error state t:
  - $\phi_3 \subseteq \neg t$;
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  - ϕ_3 is minimal;
- ϕ_3 is the second incremental lemma.

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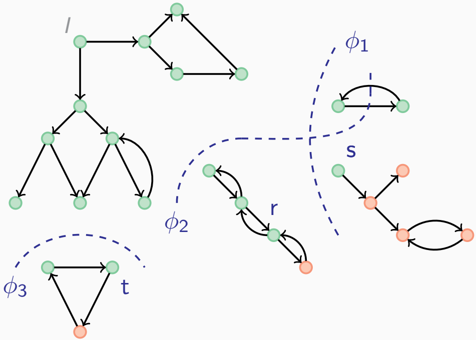

FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

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FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

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- $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P$ is an inductive strengthening.
- P is $\mathcal{M}$-invariant.

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- worst case: we proceed with the monolithic technique;

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- eventually,
  - either $I \land \neg P$ is SAT: P is not invariant;
  - or we find an inductive strengthening $P \land \bigwedge_{i=0}^{n} \phi_i$;

Complexity and Parallelization

Complexity:

- it is on the convergence of the procedure, not on the calls to the SAT-solver as before;
- each SAT-solver call is relatively small compared to those made by BMC.

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- each SAT-solver call is relatively small compared to those made by BMC.

Parallelization:

- straightforward; "by simply using a randomized decision procedure to obtain the CTIs, each process is likely to analyze a different part of the state-space." [BM07]

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- FSIS sometimes enters a long search for the next relatively inductive clauses;
- IC3 de-emphasized global information in favor of stepwise information: we will generate clauses that ensure that an error is unreachable up to some number of steps.

IC3 - Data Structures

Sequence of frames $F_0 (= I), F_1, F_2, \ldots, F_k$:

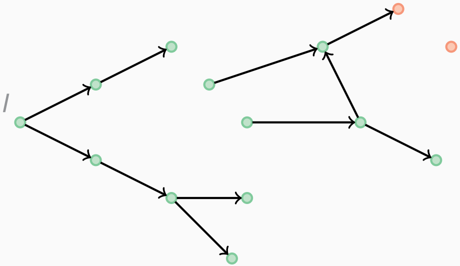
- each F_i is an over-approximation of the set of states reachable in at most k steps;
- each F_i is a set of clauses, *i.e.*, a CNF formula;
- the algorithm stops when $F_i \equiv F_{i+1}$. We will maintain the invariant that $\mathit{clauses}(F_{i+1}) \subseteq \mathit{clauses}(F_i)$: the equivalence check is simply a sintactic test: $F_i = F_{i+1}$.

IC3 - 1^{st} iteration

Check if there are counterexamples of length 0 or 1 with these two SAT-queries:

✗ $I \wedge \neg P$

✗ $F_0(= I) \wedge T \wedge \neg P'$



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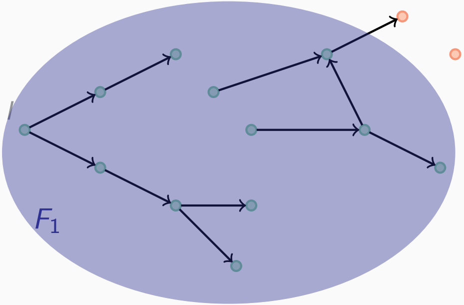
Since $F_0 \wedge T \Rightarrow P'$, we set $F_1 := P$. (over-approximation)



IC3 - 2^{nd} iteration

At iteration k, check if $F_k \wedge T \wedge \neg P'$; in this case $(k = 1)$:

$$\checkmark \quad F_1 \wedge T \wedge \neg P'$$

i.e., there exists an F_k-state that leads in one step to an error state?

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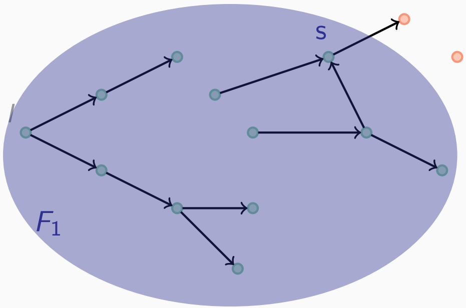

IC3 - 2^{nd} iteration (blocking phase)

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$\Rightarrow \phi_1$ excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

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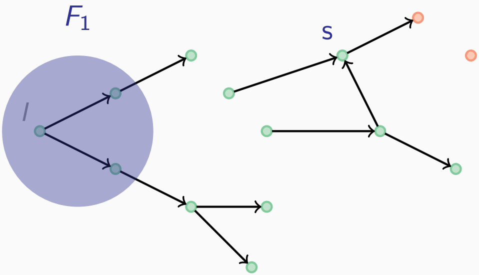
$\Rightarrow$ ϕ_1 excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

We add ϕ_1 to all the previous frames. In this case $F_1 := F_1 \wedge \phi_1$.



IC3 - Blocking phase

We have found a CTI s such that $s \models F_k \land T \land \neg P'$.

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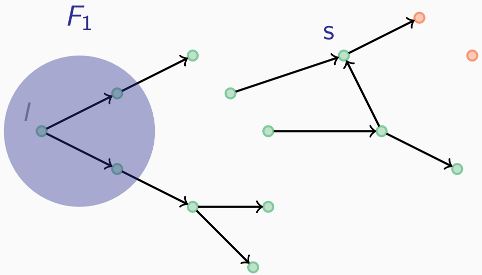
if $\neg s$ is inductive relative to F_{k-1}, then generate a minimal subclause $c \subseteq \neg s$ inductive relative to F_{k-1}, *i.e.*, c holds for at least all states reachable in i steps.

$\Rightarrow$ add c to frames $F_0 \ldots F_{k+1}$, *i.e.*, refine the over-approximations.

IC3 - 2^{nd} iteration

We create a new frame only when $F_k \land T \Rightarrow P'$ is valid.



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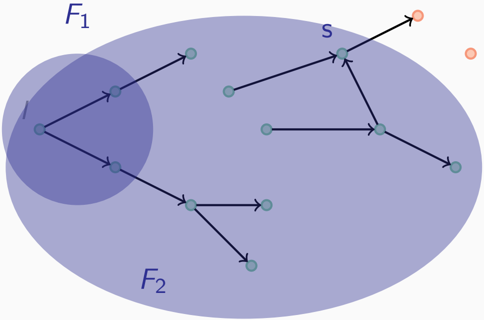
We create a new frame only when $F_k \wedge T \Rightarrow P'$ is valid.
In this case, $F_1 \wedge T \Rightarrow P'$ is valid. We create a new frame $F_2 := P$.



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For all $0 \leq i \leq k$ and $c \in F_i$, check if

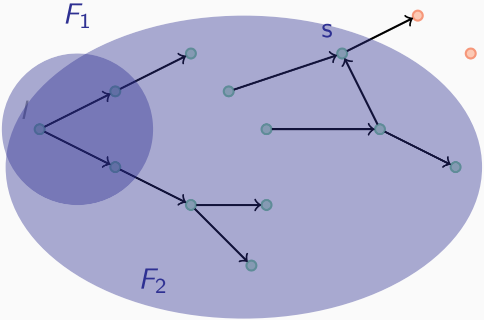
$$F_i \wedge T \Rightarrow c'$$

If $c \notin clauses(F_{i+1})$, then set $F_{i+1} \coloneqq F_{i+1} \cup \{c\}$

$\Rightarrow$ it propagates forward the errors
$\Rightarrow$ it helps the discovery of mutually inductive clauses

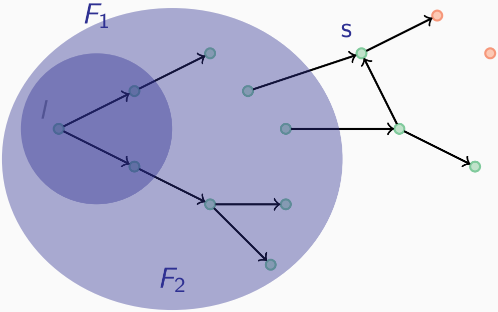
IC3 - 3^{rd} iteration

Check if $F_2 \land T \land \neg P'$ (✓). $\neg s$ is inductive relative to F_1: error state s is not reachable for at least $k = 2$ steps.



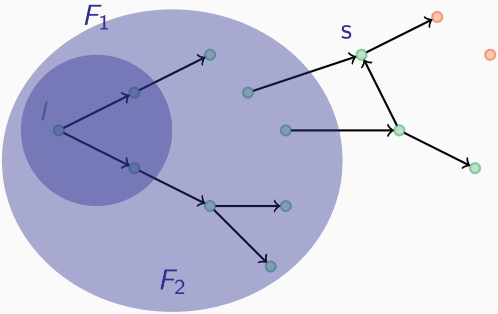
IC3 - 3^{rd} iteration

Check if $F_2 \land T \land \neg P'$ (✓). $\neg s$ is inductive relative to F_1: error state s is not reachable for at least $k = 2$ steps. Blocking phase: find minimal subclause $\phi_2 \subseteq \neg s$ inductive relative to F_1. Add ϕ_2 to frames F_0 and F_1.



IC3 - 4^{rd} iteration

Since $F_2 \wedge T \Rightarrow P'$ is valid, we create a new frame $F_3 := P$.



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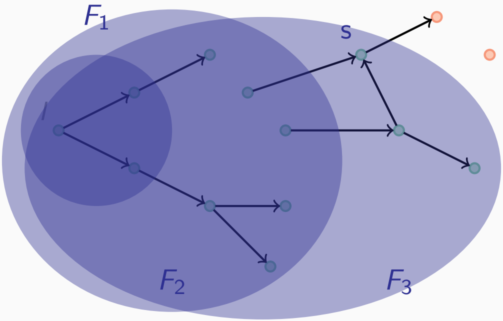
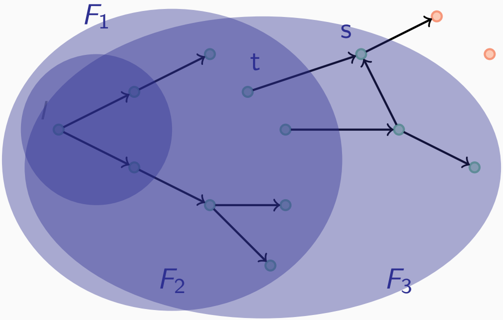


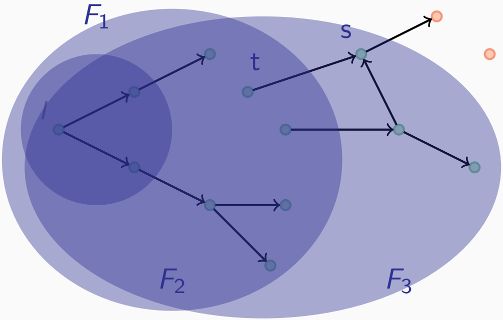
IC3 - 4^{rd} iteration

Again $F_3 \wedge T \wedge \neg P'$ (✓). But now $\neg s$ is not inductive relative to F_2



IC3 - 4^{rd} iteration

Again $F_3 \wedge T \wedge \neg P'$ (✓). But now $\neg s$ is not inductive relative to F_2: error state s could be reachable in $k = 3$ steps ...



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Instead of generating a clause that excludes s (it is possible), we call the algorithm recursively on the predecessor t of s

... remember that t could still be reachable as far as we know ...

IC3 - 4^{rd} iteration

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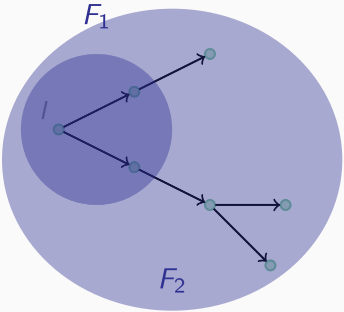
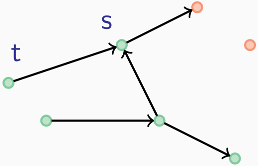
... remember that t could still be reachable as far as we know ...

"t is the new s" ;-)

IC3 - Recursion

We want to remove error state t from F_2. $\neg t$ is inductive relative to F_1: find min subclause $\phi_4 \subseteq \neg t$ and add it to F_0,F_1,F_2.





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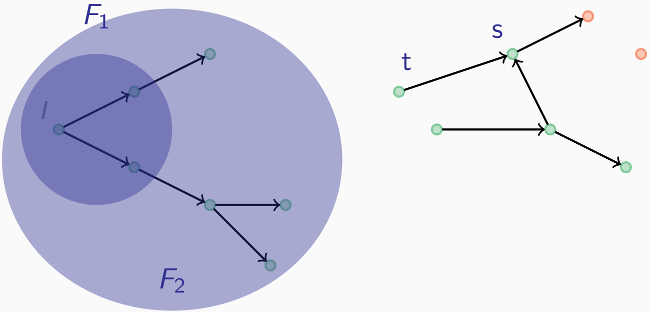
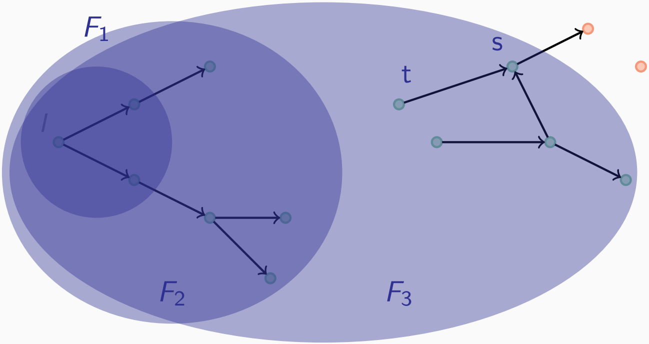


If in this process we had gone back with recursion until an initial state, then we would have found a counterexample.

IC3 - Termination

Now error state s in frame F_3 can be generalized: find min clause $\phi_5 \subseteq \neg s$ inductive relative to F_2.



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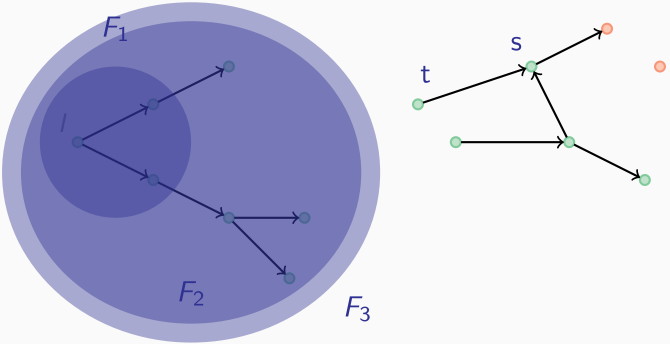

$F_2 = F_3$: IC3 terminates with True.

Beyond IC3

- FAIR: IC3 for ω-regular properties (*e.g.*, LTL) [Bra+11];
- IICTL: IC3 for CTL properties [HBS12];
- Infinite-state: software model checking via IC3 [CG12].

IC3 with nuXmv

- `go_msat` : initialise the system for infinite state model checking using the SMT solver mathsat5.
- `check_invar_ic3` : check invariant using IC3-based algorithm.
- `check_ltlspec_ic3` : check LTL using IC3-based algorithm.

But IC3 is for reachability, how do we apply it for checking LTL properties?

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