

# nuXmv: model checking timed systems

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# Timed systems

## Real time systems

- Correctness depends not only on the logical result but also on the time required to compute it.
- Common in safety-critical domains like: defense, transportation, health-care, space and avionics.

## Timed Transition System (TTS)

transitions are either discrete or time-elapses,

all clocks increase of the same amount in time-elapses.

Model checking for TTS is **undecidable**.

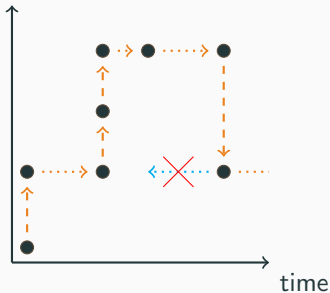
## Timed Automata (TA)

decidable restriction of TTS,

finite time abstraction:

clocks compared only to constants.

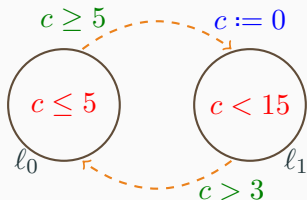
discrete



# Timed systems: representation

## Timed Automata (TA)

Explicit graph representation of discrete states (nodes) and transitions (edges).  
Symbolic representation of temporal aspects via (convex) constraints  
(**location invariants**, **transition guards** and **resets**).



## Symbolic TTS

Logical formulae represent sets of states:  $p := \{s \mid s \models p\}$ .

Transition system symbolically represented by formula  $\varphi(X, X')$ .

There is a discrete transition from  $s_0$  to  $s_1$  iff  $s_0(X), s_1(X') \models \varphi(X, X')$ .

$$l = l_0 \rightarrow c \leq 5 \quad \wedge$$

$$l = l_1 \rightarrow c < 15 \quad \wedge$$

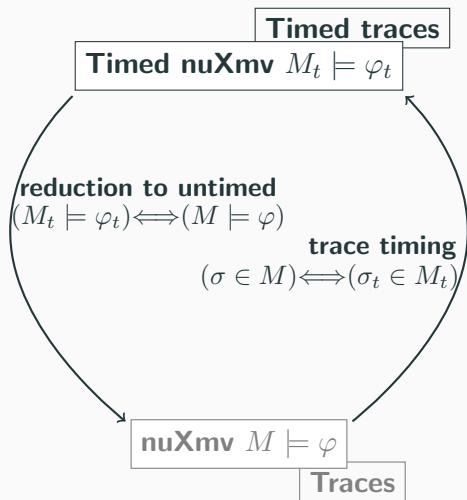
$$(l = l_1 \wedge l' = l_0) \rightarrow c > 3 \quad \wedge$$

$$(l = l_0 \wedge l' = l_1) \rightarrow (c \geq 5 \wedge c' = 0)$$

## Timed nuXmv

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# nuXmv for timed system: architecture



## Overview

- Must start with `TIME_DOMAIN continuous`;
- Symbolic description of infinite transition system using: `INIT`, `INVAR` and `TRANS` to specify initial, invariant and transition conditions.
- Model described as a synchronous composition of `MODULE` instances.
- Clock variables,
  - `time`: built-in clock variable,
  - convex invariants over clocks,
  - `URGENT`: forbid time elapse.

### Timed nuXmv adds

- `clock` variable type, all `clocks` increase of the same amount during timed transitions;
- `time`: built-in `clock`, can be used only in comparisons with constants;
- `non continuous` type modifier: symbol can change its assignment during timed transitions;
- `URGENT`: freeze time: when one of the `URGENT` conditions is satisfied only discrete transitions are allowed;
- $MTL_{0,\infty}$  specifications, by “extending” LTL;

### Timed nuXmv updates

- TRANS constrain the discrete behaviour only,
- INVAR: `clocks` allowed in invariants with shape:  
`no_clock_expr -> convex_clock_expr;`
- LTL operators:  $X, Y, U, S,$
- Bounded LTL operators.



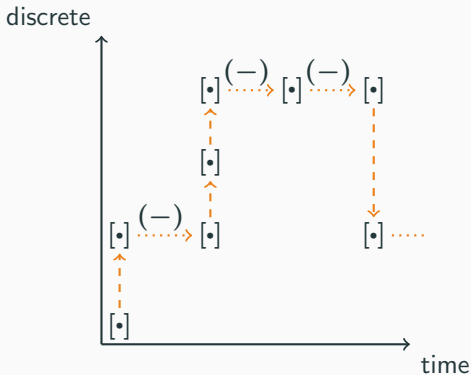
## Specification

- Different operators to refer to the *timed* next and *discrete* next:  $X$ ,  $X\sim$ ; symmetrically for the past:  $Y$ ,  $Y\sim$ .
- Time interval semantic to handle open intervals: a predicate  $p$  might hold in an interval  $(a, b]$  for  $a, b \in \mathbb{R}$ .
- Operators to retrieve value of expression the next/last time an expression will hold/held: `time_until`, `time_since`,  $@F\sim$  and  $@O\sim$ .

# Timed nuXmv: untiming

## Timed to untimed model

- `clock` symbols and `time`: variables of type `real`.
- $\delta$ : continuous positive variable, prescribes the amount of time elapse for every transition.
- $\iota$ : prescribes the alternation of singular  $[\cdot]$  and open  $(-)$  time intervals.



## Properties rewriting

**MTL** *fragment*

$$F_{[0,5]} p$$

↓ rewrite

**LTL** *timed*

$$((\neg p U p) \wedge \text{time\_until}(p) \leq 5) \vee \\ ((\neg p U \tilde{X} p) \wedge \text{time\_until}(p) < 5)$$

↓ untime

**LTL** *untimed*

$$((\neg p U p) \wedge (\text{time}@ \tilde{F} p) - \text{time} \leq 5) \vee \\ ((\neg p U ((\neg \iota \wedge p) \vee X(\neg \iota \wedge p))) \wedge (\text{time}@ \tilde{F} p) - \text{time} < 5)$$

## **Timed and infinite traces**

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# Timed and infinite traces

From untimed model execution to timed trace.

## Issue

NUXMV traces must have shape:  $\alpha\beta^\omega$ ,

$\alpha$  and  $\beta$  sequences of states.

Complete for finite state systems.

**TTS:** time monotonically increasing, infinite state system,

**undecidable.**

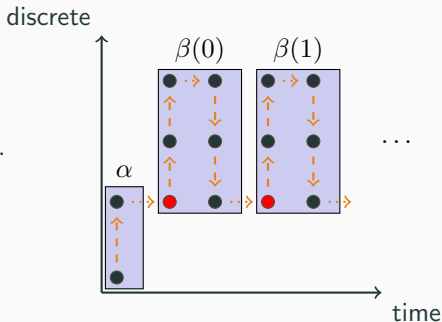
Identify traces expressible as:  $\alpha\beta(i)^\omega$ .

Same problem can be found in infinite state transition systems.

## Solution

Value assigned to variables at state  $s$  is function of the previous configuration assignments.

e.g.  $next(time) := time + \delta$



Three main operations on traces: **simulation**, **execution** and **completion**.

### Simulation

Build a possible execution of the model. The trace can be built automatically by the system or the user can choose each state from the list of possible ones.

Exploit SMT-solver to perform a discrete transition or time-elapse to obtain next configuration.

## Execution

Check if a trace belongs to the language of the model.

Exploit SMT-solver to prove that **for all** possible iterations all prescribed transition can be performed.

## Completion

A partial trace is completed so that it belongs to the model language.

Sound and complete technique requires to check if there **exists** a possible completion so that the completed trace belongs to the model language: quantifier alternation ( $\exists\forall$ ).

Adopt sound but incomplete approach.

## How to run: model [1/3]

- `./nuXmv -time -int`: start NUXMV interactively and enable commands for timed models.
- `go_time`: process the model.
- `write_untimed_model`: dump SMV model corresponding to the input timed system.



## How to run: verify [2/3]

- `timed_check_invar`: check invariants.
- `timed_check_ltlspec`: check LTL.

Mostly the same command line options of the corresponding commands for untimed models.

## How to run: simulation and traces [3/3]

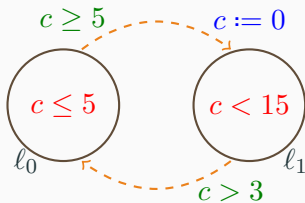
- `timed_pick_state`: pick initial state.
- `timed_simulate`: simulate the model starting from a given state.
- `execute_traces`: re-execute stored traces.
- `execute_partial_traces`: try to complete stored traces.

# Exercises

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## Simple timed automaton

Write the SMV model corresponding to the timed automaton in the figure.



### Properties

- from location  $l_0$  we always reach  $l_1$  within 5 time units;
- if we are in  $l_1$  then for the next 3 time units we remain in  $l_1$ ;
- if just arrived in  $l_1$  then for the next 3 time units we remain in  $l_1$ .

# Timed thermostat

- a thermostat has 2 states: *on* and *off*;
  - if the temperature is below 18 degrees the thermostat switches *on*.
  - if the temperature is above 18 degrees the thermostat switches *off*.
- at every time unit the temperature increases (if *on*) or decreases (if *off*) by 1;
- the thermostat measures the temperature at most ( $<$ ) every  $max\_dt$  time units.
- the temperature initially is in  $[18 - max\_dt; 18 + max\_dt]$ .

Verify that the temperature is always in  $[18 - 2max\_dt; 18 + 2max\_dt]$

# Fischer mutual exclusion protocol

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```
1: procedure FISCHER(pid, c, id)
2:   loop
3:     while id  $\neq$  0 do
4:       skip
5:       x  $\leftarrow$  random(0, c)
6:       wait_at_most(c)
7:       id  $\leftarrow$  pid
8:       wait_at_least(c)
9:       if id = pid then
10:        Critical Section
11:        id  $\leftarrow$  0
```

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Verify the mutual exclusion property.

NUXMV does not support asynchronous composition: model scheduler explicitly.